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Dynamical Aspects of Galaxy Clustering [and Discussion]

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Dynamical aspects of galaxy clustering

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Some recent work on the origin and evolution of galaxy clustering is reviewed, particularly within the context of the gravitational instability theory and the hot big-bang cosmological model. Statistical measures of clustering, including correlation functions and multiplicity functions, are explained and discussed. The close connection between galaxy formation and clustering is emphasized. Additional topics include the dependence of galaxy clustering on the spectrum of primordial density fluctuations and the mean mass density of the Universe.

1. Introduction

The distribution of galaxies on the sky manifestly is non-uniform and, by inference, the distribution in space is also non-uniform on scales up to several tens of megaparsecs and perhaps even more. The existence of this large-scale structure poses an interesting challenge to theorists: how did the present clustering pattern arise and what does it tell us about the Universe in the past? The answer to this question undoubtedly has an important bearing on theories of the origin and evolution of galaxies. What follows is a brief review of some recent observational and theoretical work on galaxy clustering and its cosmological implications. More thorough introductions to the subject can be found elsewhere (Jones 1976; Gott 1977; Rees 1978; Fall 1979; Peebles 1979).

For the sake of definiteness, the 'standard' big-bang cosmological model will be adopted throughout this discussion. That is, it will be assumed that the Universe is homogeneous in the large, that the red shifts of galaxies are the result of a general expansion and that the microwave background radiation is extragalactic and originated in a dense fireball phase of the Universe. Furthermore, it will be assumed that the expansion is governed by Friedmann's equations with the following present values of the defining parameters:

$$0.5 \lesssim h \lesssim 1.0, \quad T = 2.7\,\mathrm{K},$$
$$0.02 \lesssim \Omega \lesssim 1.0, \quad \Lambda = 0, \tag{1}$$

all in standard notation (with $h \equiv H/100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$). The range of values above is intended to reflect empirical uncertainties, except for the 'cosmological constant' Λ, which is almost completely unknown.

2. Gravitational instability

To avoid discussion of the primordial origins of structure and its evolution in the fireball plasma, the recombination epoch ($z_r \approx 1500$) will be taken as the starting point. Moreover, it will usually be assumed that on mass-scales larger than individual galaxies (*ca.* $10^{12}\ \mathrm{M_\odot}$), the fluctuations present at that time were isothermal and had a power-spectrum of the form

$$\langle|\Delta_k|^2\rangle_r \propto k^n \quad (-3 < n \leqslant 4), \tag{2}$$

where Δ_k is the Fourier transform of $\Delta(\boldsymbol{x})$, the local density contrast at the point $\boldsymbol{x}$, and n is an index to be determined empirically. For some purposes, an alternative expression in terms of mass-scales is useful:

$$\Delta_{\mathrm{r}}(M) \propto M^{-\frac{1}{2}-\frac{1}{6}n}, \tag{3}$$

where the density contrast must not be interpreted in the root-mean-square sense when averages are taken over length-scales proportional to $M^{\frac{1}{3}}$. Of course, the real fluctuation spectrum at recombination may not have been a pure power-law on all scales but (2) and (3) are at least convenient one-parameter forms to work with and seem to give reasonably good agreement with the data for $-2 \lesssim n \lesssim 0$ (depending on Ω).

As is well known, the growth rate of the gravitational instability in an expanding medium is slow. In the absence of pressure forces, the growing mode of linear theory is approximately

$$\Delta^{+} \propto \begin{cases} t^{\frac{2}{3}} & (z \gtrsim z_{\mathrm{f}}), \\ \mathrm{const} & (z \lesssim z_{\mathrm{f}}), \end{cases} \tag{4}$$

for $\Omega \lesssim 1$, where t denotes proper time from the big bang and $z_{\mathrm{f}} \approx \Omega^{-1}-1$ is the red shift when the Universe begins undecelerated expansion. Thus, the mass-scale which is just beginning nonlinear condensation varies with t as

$$M_{\mathrm{c}}(t) \propto \begin{cases} t^{4/(3+n)} & (z \gtrsim z_{\mathrm{f}}), \\ \mathrm{const} & (z \lesssim z_{\mathrm{f}}). \end{cases} \tag{5}$$

A nearly spherical inhomogeneity that grows to $\Delta \approx 1$ before z_{f} continues to expand until $\Delta \approx 5.5$, after which time it collapses and then oscillates until the equilibrium configuration is reached. An inhomogeneity that does not reach unit amplitude before z_{f} grows very little thereafter.

At any given time, fluctuations of different densities can be found in different phases of the evolutionary cycle: expansion, collapse, oscillation and equilibrium. Ignoring dissipation, the above formulae and simple energy considerations lead to some useful relations for the equilibrium configurations of structures formed by the gravitational instability mechanism. They depend on the 'initial' spectrum of fluctuations through the parameter n and, in terms of the 'final' characteristic densities, masses and sizes, take the form

$$\rho \propto M^{-\frac{3}{2}-\frac{1}{2}n} \propto R^{-(9+3n)/(5+n)}. \tag{6}$$

Another prediction of the simple theory is that the structures ought to be distributed in a nested hierarchy with small, high density systems located in large, low density systems.

3. Correlation functions

An important method for describing the large-scale matter distribution, and one that has been especially stressed by Peebles and his associates, is in terms of correlations between the positions of galaxies. In particular, the pair-correlation function ξ has received the most attention because of the relative ease with which it can be estimated empirically. This function is defined such that $\bar{n}^2(1+\xi(r))\,\delta v_1\,\delta v_2$ is the joint probability of finding galaxies in the elemental volumes δv_1 and δv_2 separated by the distance r, where $\bar{n}$ is the mean space density of galaxies. It is a measure of the relative strength of pair-wise clustering and is known to have the approximate power-law form

$$\begin{gathered} \xi(r) \approx (r_0/r)^{\gamma} \quad (0.1\,\mathrm{h}^{-1}\,\mathrm{Mpc} \lesssim r \lesssim 10\,\mathrm{h}^{-1}\,\mathrm{Mpc}), \\ \gamma \approx 1.8, \quad r_0 \approx (4 \pm 1)\,\mathrm{h}^{-1}\,\mathrm{Mpc}, \end{gathered} \tag{7}$$

except that it may also have a downward bend at $r \approx 2r_0$ (Groth & Peebles 1977). A few of the higher order correlation functions have also been estimated and they appear to have forms that are consistent with a hierarchy having characteristic densities proportional to $r^{-\gamma}$. (But whether the correlation estimates necessarily imply a hierarchical distribution is a topic of some debate: see Soneira & Peebles (1978) and Shanks (1979).)

The goal now is to relate the correlation function (7) to the 'initial' fluctuation spectrum (2). With $\Omega \approx 1$, this is relatively straightforward because most of the aggregates to which (7) refers can be presumed to have already, or to have nearly, reached the 'final' equilibrium configurations. In this case, the comparison of (7) with (6) gives

$$\gamma \approx (9+3n)/(5+n) \quad (\Omega \approx 1), \tag{8}$$

and $\gamma \approx 1.8$ therefore implies $n \approx 0$, a 'white noise' spectrum at recombination (Peebles 1974). With $\Omega \ll 1$, the problem is not nearly so simple because aggregates on scales over most of the range that ξ has been reliably estimated must be presumed to be in the expansion or collapse phases of their evolution. Indeed, fluctuations on scales larger than about r_0 should be expanding with very little deceleration, their growth having stopped at a red shift of about z_{f}.

In general, the effects of low Ω should manifest themselves in ξ as a progressive steepening of this function, but the exact form that this should take has been a matter of debate (Peebles 1974; Gott & Rees 1975; Davis *et al.* 1977). Clearly, the smaller Ω is, the more large-scale structure is required in the initial matter distribution to compensate for its slower growth rate. A simple dynamical argument that accounts for the steepening in terms of an Ω-dependent 'effective' power-law index gives (Fall 1979)

$$\gamma_{\mathrm{e}} \approx \frac{-3 \lg \Omega + \lg \xi_{\mathrm{v}}}{-\lg \Omega + \gamma_{\mathrm{f}}^{-1} \lg \xi_{\mathrm{v}}} \quad (1 \lesssim \xi \lesssim \Omega^{-3}), \tag{9}$$

where γ_{f} is given approximately by (8) and $\xi_{\mathrm{v}} \approx 300$ is roughly the density contrast that bound aggregates have when they first reach their equilibrium ('virialized') state. Although it has been tested in only a few cases, this expression is in reasonable agreement with expanding N-body experiments (section 5) and, as an example, predicts $n \approx -1.6$ for $\gamma_{\mathrm{e}} \approx 1.8$ and $\Omega \approx 0.1$. It is not expected, however, that ξ should have perfect power-law form because its index must vary from γ_{f} on small scales ($\xi \gtrsim \Omega^{-3}$) to γ_{e} on intermediate scales and then to $(3+n)$ on large scales ($\xi \lesssim 1$) or, from 1.2 (not observable) to 1.8 and then to 1.4 (possibly observable) in the above example. The remaining question is whether such behaviour is compatible with the data, and this is presently an unresolved issue.

4. Multiplicity functions

Another useful description of the large-scale matter distribution, and one that is closer to the intuitive picture of galaxy clustering, is in terms of the multiplicity function η. This function is defined such that $\eta(M, \delta)\, \mathrm{d}M$ is the mean space density of 'groups' with density contrast δ and total mass in the interval $(M, M+\mathrm{d}M)$. To make this definition precise, the notion of a group of galaxies must be specified and this is best done in terms of a 'group catalogue' $\mathscr{C}(\delta)$, defined here to be all regions of space within which the mean density contrast has the fixed value δ. Individual regions are then the groups of the catalogue $\mathscr{C}(\delta)$. The boundaries of a family of catalogues at different density contrasts make a contour map of the matter distribution and their topology indicates how hierarchical the distribution is.

In the gravitational instability theory, the dependence of $\eta(M, \delta)$ on M at fixed δ should directly reflect the initial mass-spectrum of fluctuations, independent of Ω, because Ω merely controls the relative growth rates of inhomogeneities at different densities. The dependence of $\eta(M, \delta)$ on δ at fixed M, however, should reflect both the initial spectrum and Ω in essentially the same way that they are reflected in the index of ξ. A simple set of formulae that generalize the original theory of Press & Schechter (1974) and incorporate these effects into η with the same assumptions used in the derivation of (9) is the following:

$$\left.\begin{aligned} \eta(M, \delta) &= M^{-2} G_n((M/M_1)\,\delta^{-1/\epsilon_e}), \\ G_n(x) &\propto x^{\frac{1}{2}+\frac{1}{6}n} \exp\,(-x^{1+\frac{1}{3}n}), \\ \epsilon_e &= -\gamma_e/(3-\gamma_e) \end{aligned}\right\} \tag{10}$$

(Efstathiou *et al.* 1979). Here M_1 is the current, marginally nonlinear mass-scale given by (14) below and, with $\gamma_e \approx 1.8$, the required exponent is $\epsilon_e \approx -1.5$.

Estimating the multiplicity function empirically from a large sample of galaxies is not nearly as straightforward as estimating correlation functions unless complete red shift information is available. This is because the relation between η and the projected distribution of galaxies on the sky involves several uncertain selection effects such as the apparent distance–richness correlation of groups. The estimates of Gott & Turner (1977) are based on the identification of groups as surface density enhancements greater than 8 in a 14th magnitude sample of galaxies with some supplementary red shift information for distance estimates. After various corrections, the result is

$$\eta(M, \delta_e) \underset{\sim}{\propto} \begin{cases} M^{-1.0} & (M \lesssim 8\times 10^9 h^{-2}\mu_c L_\odot), \\ M^{-2.3} & (M \gtrsim 8\times 10^9 h^{-2}\mu_c L_\odot), \end{cases} \tag{11}$$

where $\delta_e \approx 500$ is the effective space density contrast of groups in the sample and $\mu_c \approx 1600\Omega h M_\odot/L_\odot$ is the cosmic mass : light ratio. The estimates (11) do not have the functional form predicted by (10) but they do have a feature near the predicted mass scale $\delta_e^{1/\epsilon_e} M_1 \approx 8\times 10^9 h^{-2}\mu_c L_\odot$, where M_1 is given by (14) below. When complete red shift samples become available, a more detailed comparison between theory and observation should be possible.

5. Dissipative and other effects

In principle, the ideas of the previous sections can be tested against the clustering that develops in expanding N-body computer experiments. This is especially important because one might expect that various disruptive processes, such as the merging of substructures, would erase some of the hierarchical structure formed by the simple gravitational clustering mechanism. There are some technical difficulties in the interpretation of the N-body results, most of which are associated with the relatively small number of particles involved ($N \approx 10^3$); but, taken as a whole, they are in qualitative and, in some cases, quantitative agreement with the theory (Aarseth *et al.* 1978; Fall 1978; Efstathiou 1979; Efstathiou *et al.* 1979; Gott *et al.* 1979). That is, ξ and η show some dependence on Ω and n in the expected sense with no evidence for disruptive and 'bootstrap' effects at density contrasts below $\delta \approx 500$ where the observational and N-body statistics are best. At somewhat higher densities, however, there are good reasons for supposing that dissipative effects, merging and gas-dynamics, are important on small scales (White & Rees 1978).

Some indication of the effects of dissipation at high densities can be had by comparing individual galaxies with the rest of the large-scale matter distribution. Very roughly, the mass density contrast associated with the inner $10h^{-1}$ kpc of a bright galaxy is $\delta_g \approx 10^5\,\Omega^{-1}$ whereas an extrapolation of the correlation function (7) to this scale gives $(5 \pm 2) \times 10^4 h^{-1.8}$. Thus, the clustering pattern may almost match for $\Omega \approx 1$ (Peebles 1974) but $\Omega \ll 1$ implies a poor match and hence some dissipation in the development of small-scale structure. Another way to see this is in terms of the galaxy luminosity function ϕ, which is defined such that $\phi(L)\,\mathrm{d}L$ is the mean space density of galaxies with luminosities in the interval $(L, L+\mathrm{d}L)$. A group catalogue $\mathscr{C}(\delta)$ at a sufficiently high density contrast ($\delta \gtrsim \delta_g$) will include only individual galaxies and the galaxy luminosity function and the group multiplicity should be equal (Efstathiou *et al.* 1979). Thus, in any theory of galaxy formation and clustering, one has $\phi(L) = \eta(\mu_g L, \delta_g)$, where μ_g is the galactic value of M/L (roughly equal to $20hM_\odot/L_\odot$ for the inner $10h^{-1}$ kpc of bright galaxies). In a theory with very little dissipation, ϕ should have the same shape as η for all $\delta \lesssim \delta_g$ and a feature at the luminosity $(\delta_g \mu_c/\mu_g)^{1/\epsilon_e} \mu_c^{-1} M_1 \approx 1 \times 10^7 h^{-2} L_\odot$. Now Schechter (1976) has shown that the luminosities of galaxies can be fitted by a function of the form

$$\phi(L) \propto L^{-\alpha} \exp(-L/L^*), \tag{12}$$

with $L^* \approx 8 \times 10^9 h^{-2} L_\odot$ and $\alpha = 1.25$ or $\alpha \approx 1.0$ (Turner & Gott 1976). The difference between this result and the prediction of (10) and (14) indicates considerable small-scale dissipation in the formation of galaxies, perhaps of the sort discussed by White & Rees (1978).

Large-scale dissipation is an essential ingredient in the 'pancake' theory of the Moscow group (Doroshkevich *et al.* 1974). In this theory, it is supposed that the fluctuations emerging from recombination were adiabatic and had no appreciable structure on scales smaller than the Silk (1968) mass

$$M_D \approx 3 \times 10^{12}\, \Omega^{-\frac{5}{4}} h^{-\frac{5}{2}} M_\odot \tag{13}$$

as the result of photon viscosity in the fireball plasma. Structures on larger scales then grew by the ordinary gravitational instability mechanism into elongated figures while in the gaseous phase and then collapsed (at red shifts of about 3–5) to form large caustic surfaces (the 'pancakes'). Once this happened, individual galaxies and other small-scale structures are presumed to have condensed out of the gas as the result of cooling behind the shocks.

With the inclusion of gas-dynamical effects such as these, the theory and its confrontation with observation become difficult. The power-law form of the correlation function may, however, provide a simple constraint because the present, marginally linear mass-scale ($\delta \approx 1$) is about

$$M_1 \approx 8 \times 10^{14} \Omega h^{-1} (r_0/4h^{-1}\ \mathrm{Mpc})\, M_\odot \tag{14}$$

(Fall 1979). If M_D is smaller than M_1, we might reasonably expect to see some sort of feature in ξ at the corresponding spatial scale, whereas if M_D is much larger than M_1, we would not expect to find any small-scale structure (S. Bonometto & S. M. Fall 1978, unpublished). The comparison of (13) with (14) then limits the acceptable values of the density parameter to a small range around $\Omega \approx 0.08\,\mathrm{h}^{-\frac{3}{2}}$ unless it can be shown that the nonlinear gas-dynamics would have smeared out a feature in the spectrum at M_D in the available time.

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Discussion

G. Efstathiou (*Department of Physics, University of Durham, South Road, Durham DH*1 3*LE, U.K.*). Cosmological N-body simulations performed to date make the following principal assumptions: (i) radiation pressure can be ignored, (ii) zero cosmological constant, (iii) all matter is associated with galaxies, (iv) at some time after the epoch of recombination, matter was weakly clustered.

With our present knowledge of the Universe, assumptions (i), (ii) and (iii) are reasonable starting points for a discussion of galaxy clustering. Assumption (iv) is more difficult, because it is quite likely that at early times the Universe was highly nonlinear on small enough scales. Indeed, formation of clusters of galaxies by gravitational instability from a power-law spectrum of isothermal fluctuations requires nonlinear lumps of *ca.* 10^7–$10^{10}\,M_\odot$ at the epoch of recombination.

In numerical simulations, point (iv) is coupled to the fact that for small systems ($N \approx 10^3$), the inter-particle separation is a large fraction (*ca.* 0.1) of the radius of the bounding sphere. One cannot, therefore, hope to test Peebles's scaling relation for the slope of the two-point correlation function $\xi(r)$ over the full range of n and Ω by using small N-body systems.

Despite this problem, several interesting results have emerged. In the case of Poisson initial conditions ($n = 0$), the numerical simulations show that for $\Omega = 1$ the slope of $\xi(r)$ agrees reasonably well with Peebles's scaling arguments, at least on small scales. Lowering the density parameter has the effect of steepening the slope of $\xi(r)$. These results are illustrated in figures 1 and 2.

As mentioned by Fall, the slope of $\xi(r)$ is expected to be a complicated function of n and Ω and so it would be premature to draw conclusions on the value of the density parameter by using the slope of the correlation function. Before we can attempt to do so, we require more observational information on the shape of $\xi(r)$ on large scales ($> 5\,h^{-1}$ Mpc).

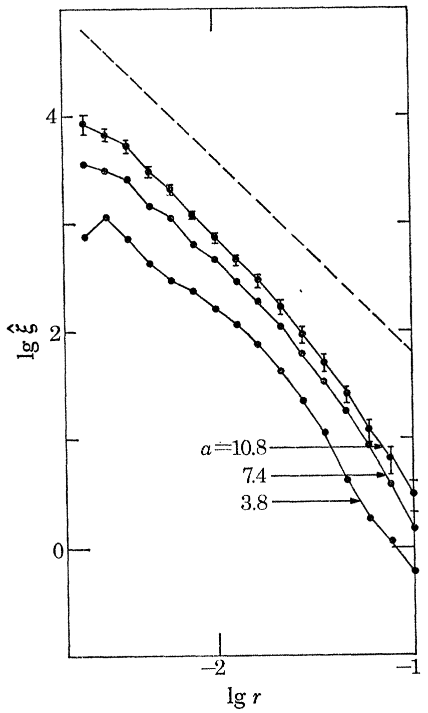

FIGURE 1. Averaged pair-correlation estimates for six 1000-body experiments with $n = 0$, $\Omega = 1.0$. The expansion parameter at which the estimates were made is denoted by a and the error bars on the last set of points are one standard deviation from the mean. The broken line is a power-law with index 1.8.

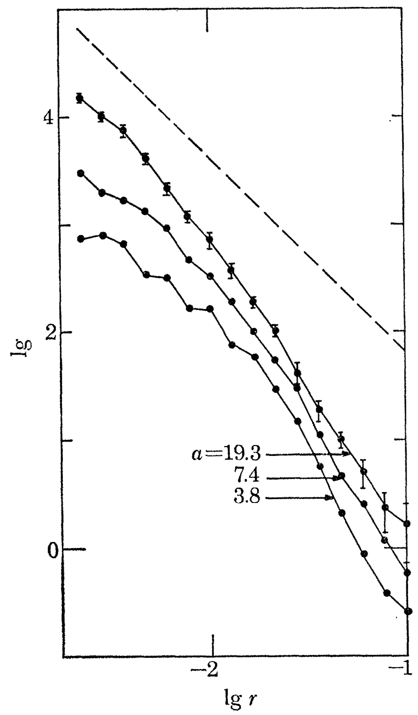

FIGURE 2. Averaged pair-correlation estimates from three 1000-body experiments with $n = 0$ and $\Omega = 0.15$ (final value). The symbols have the same meaning as in figure 1.

R. FONG (*Department of Physics, University of Durham, U.K.*). Dr Fall will, of course, be aware of the work done by Tom Shanks, who has applied a statistical test known as the Mead analysis to galaxy samples. The two- and three-point covariance functions are *just* two statistical parameters helping to characterize the distribution. Now, Shanks has found that the Mead analyses for the dynamical N-body simulations and Peebles's hierarchical model are compatible, but differ significantly from the Mead analysis of observed galaxy samples, such as the Zwicky catalogue and our own deep galaxy samples. Is this not a rather disconcerting result, if we are to hope that dynamical N-body simulations and the gravitational instability theory of the formation of galaxies from isothermal perturbations give us an understanding of the formation and evolution of the distribution of galaxies?

S. M. FALL. Taken at face value, it would seem that Shanks's work does favour an adiabatic picture for the formation of galaxies and clusters of galaxies. This is a subtle problem, however, and some recent work of Peebles suggests that the two- and three-point correlation functions are only compatible with a hierarchical distribution of galaxies and therefore with the isothermal picture. This contradicts some of Shanks's work but it does not answer his point about the Mead statistic, so the situation is unclear.

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